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FORMULATION OF MODE COUPLING EQUATIONS AT STEP DISCONTINUITY BASED ON THE PLANAR CIRCUIT THEORY

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1.Introduction

Surface-wave planar circuits are frequently used at microwave, millimetric and optical waves. In order to construct a practical surface-wave planar circuit as shown in Fig.1, side-wall is needed to reflect or confine the surface-wave laterally. Usually two kinds of side-wall,i.e. metal wall or total power reflection wall above the critical angle(effected by step discontinuity as shown in Fig.2) are utilized. The former is free from mode conversion, and conveniently used at microwaves, but not so at optical and millimetric waves because of the Ohmic loss. On the other hand the latter is useful at optical and millimetric waves because it is free from the Ohmic loss.

However the discontinuity itself disturbs the electromagnetic field near the step, which causes the excitation of higher modes including continuous spectrum and the other type of modes (TE-TM mode conversion) at both sides of the step. These excited modes may be either evanescent or propagating, and may introduce some reactive loading or radiation loss at the side-wall of the planar circuit ,which sometimes shift or deteriorate the desired operation of the planar circuit itself. Therefore it is meaningful and useful to provide a circuit theory which can treat the above mentioned problems exactly and systematically.

In this paper notifying this importance, general mode coupling equations at the straight uniform step discontinuity as shown in Fig.2 are derived with properly defined mode voltage and current, and formulated in matrix forms. Through these derivations mode voltage and mode current are defined according to the planar circuit theory, and the consistency with the planar circuit theory and the conventional circuit theory are kept as much as possible. Next, it is shown that derived general mode coupling equations can be easily applied to the analysis of the practical structure.

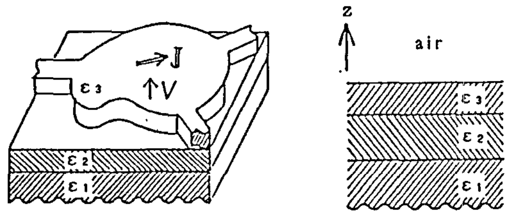
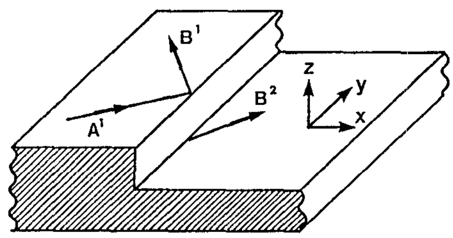


Fig.1 Surface-wave planar circuit.



region 1 region 2

Fig.2 Straight uniform step discontinuity.

2.Description of the electromagnetic field in the planar circuit with mode voltage and current

Before deriving the mode coupling equations at the step discontinuity,the field in the surface-wave planar circuit is described by virtue of the appropriately defined mode voltage and two dimensional vector mode current.

Suppose that the surface-wave planar circuit has a uniform structure in x-y plane and its specific dielectric constant $\varepsilon_s(z)$ is only a function of the height coordinate z as shown in Fig.1. There exists an infinite number of independent basic TE and TM surface-wave modes, whose functional form in the height direction f(z),g(z) and h(z) are given by (G),(H) and(I) in Table 1. Here "n" means mode number and the suffix "E" or "H" means mode TM or TE.

When mode voltage Vn(x,y) and vector mode current Jn(x,y) are given by solving planar circuit equations (D) and (E) in Table 1, then the fields component of the each mode in the planar circuit is expressed by eq.(A),(B),(C) in Table 1. Hence the total field can be described by the following equations.

$$E_z(x,y,z)=-\Sigma V_n^E(x,y)\, g_n^E(z)$$

$$H_z(x,y,z)=-\Sigma V_n^H(x,y)\, g_n^H(z)$$

$$E_t(x,y,z)=\Sigma\,[J_n^H(x,y)\times k]\, f_n^H(z) + j\eta_0 \Sigma J_n^E(x,y) h_n^E(z) \quad (1)$$

$$H_t(x,y,z)=j\eta_0^{-1}\,\Sigma J_n^H(x,y) h_n^H(z) + \Sigma\,[k\times J_n^E(x,y)]\, f_n^E(z)$$

The voltage and current of each mode in the uniform planar circuit can be given by eqs.(2)

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$$V(x,\ y)=\Sigma A(\beta_y)\, e^{-j(\beta_x\cdot x+\beta_y\cdot y)}$$

$$J(x,\ y)=\Sigma Y_c \frac{\beta_t}{\beta_t} A(\beta_y)\, e^{-j(\beta_x\cdot x+\beta_y\cdot y)} \qquad (2)$$

as a solution of two dimensional Helmholtz equation.

$$\nabla_t^2 V + \beta_t^2 V = 0$$

where $\beta_t=\sqrt{XB}$ is the planar propagation constant and Y_c is planar characteristic admittance of the corresponding mode.

3. Derivation and Formulation of Mode coupling equations

When the planar circuit has no discontinuity, there is no mode coupling. However when there exists a step discontinuity as shown in Fig.2, coupling between the modes in each region occurs,as is well known. In order to treat these problems quantitatively and systematically, coordinate system as shown in Fig.2 and 4 is supposed; the suffix "1" or "2" is used to describe the physical quantity in connection with the region 1 (x<0) or the region 2 (x>0) respectively; also the symbol "$\perp$" or "$/\!/$" is used instead of x or y in order to describe the physical situation more clearly. We know that only the modes which share the same $\beta_y=\beta_{/\!/}$ can couple at the step discontinuity. Hence, the coupled mode voltage and current in each region can be expressed by eqs.(3) instead of eqs.(2)

$$V(x,\ y)=(A\, e^{-j\beta_\perp\cdot x}+B\, e^{j\beta_\perp\cdot x})\, e^{-j\beta_{/\!/}\cdot y}$$

$$J_\perp(x,\ y)=Y_{c\perp}\ (A\, e^{-j\beta_\perp\cdot x}-B\, e^{j\beta_\perp\cdot x})\, e^{-j\beta_{/\!/}\cdot y}$$

$$J_{/\!/}(x,\ y)=Y_{c/\!/}\ (A\, e^{-j\beta_\perp\cdot x}+B\, e^{j\beta_\perp\cdot x})\, e^{-j\beta_{/\!/}\cdot y} \qquad (3)$$

where, $Y_{c\perp}=Y_c(\beta_\perp/\beta_t)$ $\quad Y_{c/\!/}=Y_c(\beta_{/\!/}/\beta_t)$

$$\beta_\perp=\sqrt{\beta_t^2-\beta_{/\!/}^2} \qquad \beta_t=\beta_n$$

At the step discontinuity, the tangential components of the field of both side must always be continous. Hence, the following equations must hold.

$$Hz: \Sigma V_n^{1H} g_n^{1H}(z)=\Sigma V_n^{2H} g_n^{2H}(z)$$

$$Ez: \Sigma V_n^{1E} g_n^{1E}(z)=\Sigma V_n^{2E} g_n^{2E}(z)$$

$$Ey: -\Sigma \vec{J}_{n\perp}^{1H} f_n^{1H}(z)+j\eta_0\Sigma J_{n/\!/}^{1E} h_n^{1E}(z)= -\Sigma \vec{J}_{n\perp}^{2H} f_n^{2H}(z)+j\eta_0\Sigma J_{n/\!/}^{2E} h_n^{2E}(z) \qquad (4)$$

$$Hy: \Sigma \vec{J}_{n\perp}^{1E} f_n^{1E}(z)-\frac{1}{j\eta_0}\Sigma J_{n/\!/}^{1H} h_n^{1H}(z)= +\Sigma \vec{J}_{n\perp}^{2E} f_n^{2E}(z)-\frac{1}{j\eta_0}\Sigma J_{n/\!/}^{2H} h_n^{2H}(z)$$

Applying the orthogonality relations shown in (I) of Table 1 to eqs.(4), following mode coupling equations are derived.

$$V_m^{2H} = \Sigma F_{(n,m)}^{1H2H} V_n$$

$$V_m^{2E} = \Sigma F_{(n,m)}^{1E2E} V_n$$

$$\vec{J}_{m\perp}^{1H} - \Sigma F_{(n,n)}^{1H2H} \vec{J}_{n\perp}^{2H} = j\eta_0 \{\Sigma H_{(m,n)}^{1H1E} J_{n/\!/}^{1E} - \Sigma H_{(m,n)}^{1H2E} J_{n/\!/}^{2E}\}$$

$$\vec{J}_{m\perp}^{1E} - \Sigma F_{(m,n)}^{1E2E} \vec{J}_{n\perp}^{2E} = \frac{1}{j\eta_0} \{\Sigma H_{(m,n)}^{1E1H} J_{n/\!/}^{1H} - \Sigma H_{(m,n)}^{1E2H} J_{n/\!/}^{2H}\} \qquad (5)$$

where mode coupling coefficients are defined as follow.

$$F_{(m,n)}^{1H2H} = \langle g_m^{1H}, f_n^{2H}\rangle \qquad F_{(m,n)}^{1E2E} = \langle g_m^{1E}, f_n^{2E}\rangle$$

$$H_{(m,n)}^{1H1E} = \langle g_m^{1H}, h_n^{1E}\rangle \qquad H_{(m,n)}^{1E1H} = \langle g_m^{1E}, h_n^{1H}\rangle \qquad (6)$$

$$H_{(m,n)}^{1H2E} = \langle g_m^{1H}, h_m^{2E}\rangle \qquad H_{(m,n)}^{1E2H} = \langle g_m^{1E}, h_m^{2H}\rangle$$

$$\langle a,b\rangle \equiv \int a(z)\cdot b(z)\,dz$$

TABLE 1 Fundamental relation of surface-wave planar circuit.

	TE (H) MODE	TM (H) MODE	
Field component	$Hz(x,y,z) \equiv -V^H{}_n(x,y)\cdot g^H{}_n(z)$	$Ez(x,y,z) \equiv -V^E{}_n(x,y)\cdot g^E{}_n(z)$	A
	$Et(x,y,z) \equiv [J^H{}_n(x,y)\times k]\cdot f^H{}_n(z)$	$Ht(x,y,z) \equiv [k\times J^E{}_n(x,y)]\cdot f^E{}_n(z)$	B
	$Ht(x,y,z) \equiv j\dfrac{1}{\eta_o} J^H{}_n(x,y)\cdot h^H{}_n(z)$	$Et(x,y,z) \equiv j\eta_o J^E{}_n(x,y)\cdot h^E{}_n(z)$	C
Planar circuit equations	$\mathrm{grad}V^H{}_n=-Z^H{}_n J^H{}_n$, $Z^H{}_n=j\dfrac{(\beta^H{}_n)^2}{\omega\mu}$ [S]	$\mathrm{grad}V^E{}_n=-Z^E{}_n J^E{}_n$, $Z^E{}_n=j\dfrac{(\beta^E{}_n)^2}{\omega\varepsilon_o}$ [Ω]	D
	$\mathrm{div}J^H{}_n=-Y^H{}_n V^H{}_n$, $Y^H{}_n=j\omega\mu$ [Ω]	$\mathrm{div}J^E{}_n=-Y^E{}_n V^E{}_n$, $Y^E{}_n=j\omega\varepsilon_o$ [S]	E
	$\beta_t=\beta^H{}_n$, $Z^H{}_{cn}=\dfrac{\beta^H{}_n}{\omega\mu}$ [S] , $Y^H{}_{cn}=\dfrac{\omega\mu}{\beta^H{}_n}$ [Ω]	$\beta_t=\beta^E{}_n$, $Z^E{}_{cn}=\dfrac{\beta^E{}_n}{\omega\varepsilon_o}$ [Ω] , $Y^E{}_{cn}=\dfrac{\omega\varepsilon_o}{\beta^E{}_n}$ [S]	F
Basic surface-wave	$\dfrac{d^2g^H{}_n}{dz^2}+[k_o{}^2\varepsilon_{s(z)}-(\beta^H{}_n)^2]\,g^H{}_n=0$	$\dfrac{d}{dz}[\dfrac{1}{\varepsilon_{s(z)}}\dfrac{d}{dz}\{\varepsilon_{s(z)}g^E{}_n\}]+[k_o{}^2\varepsilon_{s(z)}-(\beta^E{}_n)^2]\,g^E{}_n=0$	G
	$f^H{}_n=g^H{}_n$, $h^H{}_n=\dfrac{1}{k_o}\dfrac{df^H{}_n}{dz}$	$f^E{}_n=\varepsilon_{s(z)}g^E{}_n$, $h^E{}_n=\dfrac{1}{k_o}\dfrac{1}{\varepsilon_{s(z)}}\dfrac{df^E{}_n}{dz}$	H
	$\langle g^H{}_m, f^H{}_n\rangle \equiv \int g^H{}_m f^H{}_n dz' = \delta_{mn}$	$\langle g^E{}_m, f^E{}_n\rangle \equiv \int g^E{}_m f^E{}_n dz = \delta_{mn}$	I

k: unit vector toward height direction $\qquad k_o=\omega\sqrt{\varepsilon_o\mu}$, $\eta_o=\sqrt{\mu/\varepsilon_o}$

In order to show the mode coupling situation more clearly, we define the the following matrix.

mode voltage and mode current column matrices

$$\mathbb{V}^{1H}=(V_1^{1H},\ V_2^{1H},\ V_3^{1H},\ \cdots)^t$$
$$\mathbf{J}_\perp^{1H}=(J_{1\perp}^{1H},\ J_{2\perp}^{1H},\ J_{3\perp}^{1H},\ \cdots)^t \qquad (7)$$
$$\mathbf{J}_{/\!/}^{1H}=(J_{1/\!/}^{1H},\ J_{2/\!/}^{1H},\ J_{3/\!/}^{1H},\ \cdots)^t$$

$\mathbb{V}^{1H}$, $\mathbb{V}^{2H}$, $\mathbb{V}^{2E}$, $\mathbf{J}_\perp^{1E}$, $\mathbf{J}_\perp^{2H}$, $\mathbf{J}_\perp^{2E}$, $\mathbf{J}_{/\!/}^{1E}$, $\mathbf{J}_{/\!/}^{2H}$, $\mathbf{J}_{/\!/}^{2E}$ are defined in the same way.

mode coupling matrices

$$\mathbb{F}^{1H,2H}=\langle F_{m,n}^{1H,2H}\rangle \qquad \mathbb{F}^{1E,2E}=\langle F_{m,n}^{1E,2E}\rangle$$
$$\mathbb{H}^{1H,1E}=\langle H_{m,n}^{1H,1E}\rangle \qquad \mathbb{H}^{1E,1H}=\langle H_{m,n}^{1E,1H}\rangle \qquad (8)$$
$$\mathbb{H}^{1H,2E}=\langle H_{m,n}^{1H,2E}\rangle \qquad \mathbb{H}^{1E,2H}=\langle H_{m,n}^{1E,2H}\rangle$$

planar characteristics admittance matrices

$$\mathbf{Y}_{c/\!/}^{1H}=\mathrm{diag}\,(Y_{c1/\!/}^{1H},\ Y_{c2/\!/}^{1H},\ Y_{c3/\!/}^{1H},\ \cdots)$$
$$\mathbf{Y}_{c\perp}^{1H}=\mathrm{diag}\,(Y_{c1\perp}^{1H},\ Y_{c2\perp}^{1H},\ Y_{c3\perp}^{1H},\ \cdots) \qquad (9)$$
$$\mathbf{Y}_{c}^{1H}=\mathrm{diag}\,(Y_{c1}^{1H},\ Y_{c2}^{1H},\ Y_{c3}^{1H},\ \cdots)$$

$\mathbf{Y}_c^{1E}$ $\mathbf{Y}_c^{2H}$ $\mathbf{Y}_c^{2E}$ are defined in the same way.

Then the mode coupling equations can be summarized in Table 2. In this Table (A)and (B) means current conservation Law of TE and TM,where righthand side terms mean the current generated by the TE-TM mode conversion at the discontinuity. (C) and (D) mean ideal transformers. (E)~(H) mean that the parallel current are alway propotional to the voltage at that point and the propotional constant is the parallel characteristics admittance Eqs.(A) and (B) in Table 2 can be transformed to eq.(10),which is just parallel with eqs.(C) and eq.(D) in Table 2

$$\vec{\mathbf{J}}_\perp^{1H}-\mathbb{H}^{1H,1E}\mathbf{J}_{/\!/}^{1E}=\mathbb{F}^{1H,2H}(\vec{\mathbf{J}}_\perp^{2H}-\mathbb{H}^{2H,2E}\mathbf{J}_{/\!/}^{2E}) \qquad (10)$$
$$\vec{\mathbf{J}}_\perp^{1E}-\mathbb{H}^{1E,1H}\mathbf{J}_{/\!/}^{1H}=\mathbb{F}^{1E,2E}(\vec{\mathbf{J}}_\perp^{2E}-\mathbb{H}^{2E,2H}\mathbf{J}_{/\!/}^{2H})$$

Table 2 Mode coupling equations.

$\vec{\mathbf{J}}_\perp^{1H}-\mathbb{F}^{1H,2H}\vec{\mathbf{J}}_\perp^{2H}=j\eta_0\ \{\mathbb{H}^{1H,1E}\mathbf{J}_{/\!/}^{1E}-\mathbb{H}^{1H,2E}\mathbf{J}_{/\!/}^{2E}\}$	(A)
$\vec{\mathbf{J}}_\perp^{1E}-\mathbb{F}^{1H,2E}\vec{\mathbf{J}}_\perp^{2E}=\frac{1}{j\eta_0}\ \{\mathbb{H}^{1E,1H}\mathbf{J}_{/\!/}^{1E}-\mathbb{H}^{1E,2H}\mathbf{J}_{/\!/}^{2H}\}$	(B)
$\mathbb{V}^{2H}=(\mathbb{F}^{1H2H})^t\ \mathbb{V}^{1H}$	(C)
$\mathbb{V}^{2E}=(\mathbb{F}^{1E2E})^t\ \mathbb{V}^{1E}$	(D)
$\mathbf{J}_{/\!/}^{1H}=\mathbf{Y}_{c/\!/}^{1H}\mathbb{V}^{1H}$	(E)
$\mathbf{J}_{/\!/}^{1E}=\mathbf{Y}_{c/\!/}^{1E}\mathbb{V}^{1H}$	(F)
$\mathbf{J}_{/\!/}^{2H}=\mathbf{Y}_{c/\!/}^{2H}\mathbb{V}^{2H}$	(G)
$\mathbf{J}_{/\!/}^{2E}=\mathbf{Y}_{c/\!/}^{2E}\mathbb{V}^{2E}$	(H)

4.Effective mode admittance of the side-wall

The step discontinuity shown in Fig.2 can be modelled as shown in Fig.3.

Effective mode admittance of the side wall, i.e. mode admittance looking into the right direction at reference plane(1) is derived from the mode coupling equations in Table 2 and is given by eq.(11).

$$\begin{vmatrix}\mathbf{J}_\perp^{1H}\\ \mathbf{J}_\perp^{1E}\end{vmatrix}=\begin{vmatrix}\mathbf{Y}_{eff}^{1H1H} & \mathbf{Y}_{eff}^{1H1E}\\ \mathbf{Y}_{eff}^{1E1H} & \mathbf{Y}_{eff}^{1E1E}\end{vmatrix}\begin{vmatrix}\mathbb{V}^{1H}\\ \mathbb{V}^{1E}\end{vmatrix} \qquad (11)$$

where

$$\mathbf{Y}_{eff}^{1H1H}=\mathbb{F}^{1H2H}\mathbf{Y}_{c\perp}^{2H}\ (\mathbb{F}^{1H2H})^t$$
$$\mathbf{Y}_{eff}^{1E1E}=\mathbb{F}^{1E2E}\mathbf{Y}_{c\perp}^{2E}\ (\mathbb{F}^{1E2E})^t$$
$$\mathbf{Y}_{eff}^{1H1E}=j\eta_0\ \{\mathbb{H}^{1H1E}\mathbf{Y}_{c/\!/}^{1E}-\mathbb{H}^{1H2E}\mathbf{Y}_{c/\!/}^{2E}\ (\mathbb{F}^{1E2E})^t\}$$
$$\mathbf{Y}_{eff}^{1E1H}=\frac{1}{j\eta_0}\ \{\mathbb{H}^{1E1H}\mathbf{Y}_{c/\!/}^{1H}-\mathbb{H}^{1E2H}\mathbf{Y}_{c/\!/}^{2H}\ (\mathbb{F}^{1H2H})^t\}$$

When column matrices of TE mode and TM mode are combined like $\mathbf{V}=(\mathbf{V}^H,\ \mathbf{V}^E)$, $\vec{\mathbf{J}}_\perp=(\mathbf{J}_\perp^H,\ \mathbf{J}_\perp^E)$, then the mode voltage and perpendicular mode current can be described by the following equation by virtue of the admittance $\mathbf{Y_W}$ defined in by eq.(11).

$$\vec{\mathbf{J}}_\perp^{(1)}=\mathbf{Y_W}\ \mathbb{V}^{(1)} \qquad (12)$$

Equivalent circuit is shown in Fig.5(a), which is just the same with Fig.3.

When the side-wall works as the total reflection wall,all elements of $\mathbf{Y_W}$ are imaginary. $\mathbf{Y}_{eff}^{1E1H}$ and $\mathbf{Y}_{eff}^{1H1E}$ means effective TE-TM mode coupling admittance at the step. The effective wall admittance is a useful concept for the analysis of the planar structure.

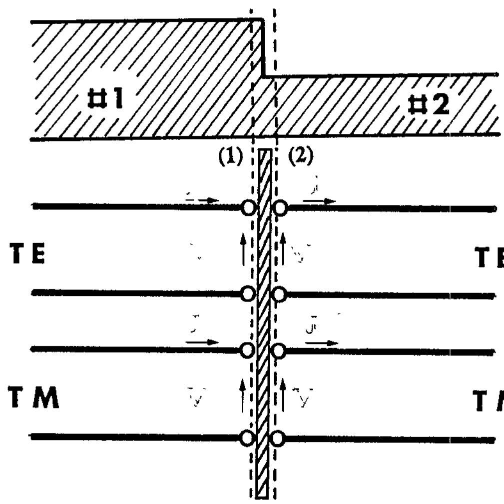

Fig.3 Circuit model of step discontinuity.

5. Practical Applications

Derived general mode coupling equations can be easily applied to the analysis of the following cases.

A. Scattering of obliquely incident surface-wave at the step

When surface-waves $A^{(1)}$ are incident from region 1 as shown in Fig.2 or 4, the reflected waves can be calculated by eq.(13)

$$B^{(1)} = (Y_{c\perp}^{(1)} + Y_W)^{-1} (Y_{c\perp}^{(1)} - Y_W) A^{(1)} \qquad (13)$$

where Y_W is wall admittance looking into the right direction from the right side of region 1. Once $B^{(1)}$ are calculated from eq.(13), the refracted waves are given by eq.(14)

$$B^{(2)} = (F^{1,2})^t (A^{(1)} + B^{(1)}) \qquad (14)$$

$$F^{1,2} = \begin{vmatrix} F^{1H2H} & 0 \\ 0 & F^{1E2E} \end{vmatrix}$$

B. Planar waveguide

The planar waveguide has two steps discontinuity as shown in Fig.5(b). When each wall admittance are given by $Y_W^{(1)}$ and $Y_W^{(2)}$, and column voltages at port(1) and port(2) are given by $V^{(1)}$ and $V^{(2)}$, the following equations must hold.

$$\begin{vmatrix} -Y_{c\perp}\cot\beta_\perp W + Y_W^{(1)} & jY_{c\perp}\operatorname{cosec}\beta_\perp W \\ jY_{c\perp}\operatorname{cosec}\beta_\perp W & -Y_{c\perp}\cot\beta_\perp W + Y_W^{(2)} \end{vmatrix} \begin{vmatrix} V^{(1)} \\ V^{(2)} \end{vmatrix} = 0 \qquad (15)$$

By solving these eigenvalue equations, the propagation constant and field distributions of the waveguide can be obtained.

C. Parallel coupled planar waveguide

The parallel coupled planar waveguide usually has symmetry at the center as shown in Fig.5(c). Hence, the analysis is performed for the one-half. The wall admittance at(1) is already given by eq.(11). $Y_W^{(2)}$ is given by eq.(16).

$$Y_W^{(2)} = \begin{vmatrix} F^{1H3H} Y_{c\perp}^{3H} \alpha (F^{1H3H})^t & j\eta_0 \{H^{1H1E} Y_{c\parallel}^{1E} - H^{1H3E} Y_{c\parallel}^{3E} (F^{1E3E})^t\} \\ \frac{1}{j\eta_0}\{H^{1E1H} Y_{c\parallel}^{1H} - H^{1E3H} Y_{c\parallel}^{3H} (F^{1H3H})^t\} & F^{1E3E} Y_{c\perp}^{3E} \alpha^{-1} (F^{1E3E})^t \end{vmatrix} \qquad (16)$$

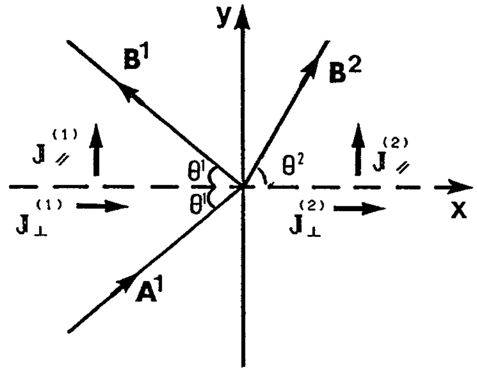

Fig.4 Reflection and refraction at step discontinuity

where $\alpha = j\tan\beta_\perp s/2$ for electric wall; $\alpha = -j\cot\beta_\perp s/2$ for magnetic wall. Once $Y_W^{(1)}$ and $Y_W^{(2)}$ are given, the eigenvalue equation is same with eq.(15). Even when the structure in Fig.5 loses its symmetry, the same reasoning can be applied and propagation constant and the field distribution can be obtained in the same way.

6. Conclusion

General mode coupling equations at step discontinuity are derived and formulated based on the planar circuit theory. Henceforth, we will use these relations for the analysis and calculation of the properties of many kinds of planar structures.

Reference

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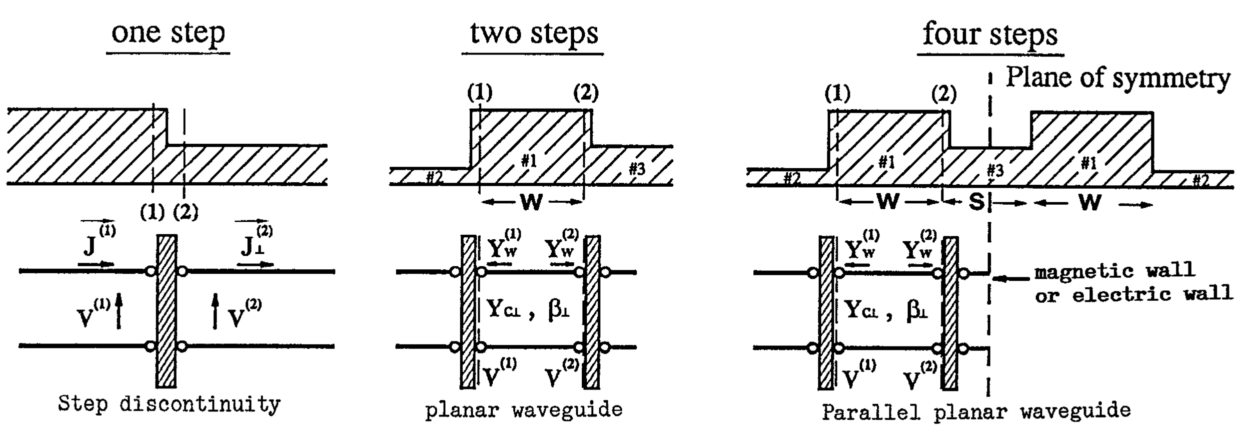

Fig.5 Structure and model of planar waveguide